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Methods and Means of Anti-Submarine Attack

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ABSTRACT

This report aims at reducing the percentage of failures in anti-submarine attack by eliminating so far as possible the element of chance. It discusses and evaluates the various factors which determine the number of bombs needed to assure success, and suggests practical ways of reducing this number to an extent that will permit destroyers to utilize direct high speed attacks.

It points out the immediate necessity of increasing the attacking speed of destroyers, of providing continuous listening and echo contact throughout the entire attack, and of greatly increasing the speed of fall of depth charges.

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INTRODUCTION

1. According to available statistics the rate of sinking of ship tonnage by aircraft, mines, and submarines is decidedly greater than the present possible rate of replacement by new construction, and approximately half of this loss is credited to the submarine. Moreover, these statistics show that present methods and means of anti-submarine attack are lacking in effectiveness, since of the attacks made only about one in ten is successful. This grave situation both justifies and calls for an analysis of the whole anti-submarine problem directed toward outlining the responsibility that rests on each of the agencies involved in its solution.

2. The only practical means thus far devised for destroying a submerged submarine are bombs designed to explode at a predetermined depth, or upon contact with the submarine's hull. If properly placed, one bomb of either type is sufficient to put the submarine out of commission. But since, according to present tactics, the location of the submarine is indefinite both as regards vertical and horizontal directions, an attack involves the laying of several bombs in a predetermined pattern to improve the chances of making a hit. Thus it may be noted that the success or failure of an attack is a matter of chance, and that statistics indicate the present chances of success are about one in ten.

3. The anti-submarine problem will be properly solved when, and only when, the element of chance is practically eliminated from the attack. Theoretically this can be accomplished by determining the volume beyond which the submarine cannot escape during an attack, and filling this volume with a proper pattern of bombs. The mathematical expressions of these conditions serve to evaluate the relative importance of each of the several factors involved and thus indicate the way toward improvement.

THEORETICAL NUMBER OF DEPTH CHARGES REQUIRED

4. The volume (V_o) beyond which the submarine cannot escape during a depth charge attack is determined by her maximum depth of submergence (H), the range at which echo contact is lost (L), the speed of the attacking destroyer (S_d), the maximum submerged speed of the submarine (S_s), the minimum tactical turning radius of the submarine (R), and the velocity of fall of the bomb (S_b). Assuming that the destroyer reaches the submarine by proceeding from the point where the echo was lost at a distance L, and thence on a distance U which terminates at the end of the depth charge pattern, the maximum distance (U) which the submarine can cover after echo contact is lost is shown on Plate 1 to be:

$$U = \frac{S_d\, S_s}{S_d - S_s} \left[\frac{L}{S_d} + \frac{H}{S_b} \right] \qquad (1)$$

Assuming no knowledge of the submarine's direction, it follows that the horizontal area A_o, a circle at depth H, beyond which she cannot escape becomes:

$$A_o = \pi U^2 = \pi \left\{ \frac{S_d \, S_s}{S_d - S_s} \left[\frac{L}{S_d} + \frac{H}{S_b} \right] \right\}^2 \tag{2}$$

Thus the operating volume (V_o) becomes:

$$\begin{aligned} V_o &= \int_0^H A_o \, dH \\ &= \frac{\pi}{3} \left(\frac{S_d \, S_s}{S_d - S_s} \right)^2 \left(3 \frac{HL^2}{S_d^2} + 3 \frac{H^2 L}{S_d S_b} + \frac{H^3}{S_b^2} \right) \end{aligned} \tag{3}$$

5. The minimum number of depth charges (N) required to make sure of a successful attack is given by dividing V_o by the destructible volume of the submarine, which is defined as the volume enclosed by a surface representing the locus of a point at distance B outside of the submarine, when B represents the destructive range of the bomb. If, then, we represent the form of the submarine as an ellipsoid of revolution with length 2a and beam 2b, its destructible volume (V_{des}) becomes:

$$V_{des} = \frac{4}{3} \pi (a + B)(b + B)^2, \tag{4}$$

and, therefore, N becomes:

$$\begin{aligned} N &= \frac{V_o}{V_{des}} \\ &= \frac{\left(\frac{S_d \, S_s}{S_d - S_s} \right)^2 \left(3 \frac{HL^2}{S_d^2} + 3 \frac{H^2 L}{S_d S_b} + \frac{H^3}{S_b^2} \right)}{4 \, (a + B) \, (b + B)^2} \end{aligned} \tag{5}$$

Obviously the solution of this equation should give the minimum number of depth charges needed to assure success in all cases when neither the speed or the direction of the submarine's course is known.

6. Assume now that the speed of the submarine is known with some accuracy at the time echo contact is lost, and that her speed during the attack proves to be something less than her maximum (S_s). No change is necessary in Equation 5 other than the simple substitution of the actual speed for the maximum speed previously assumed.

7. Finally assume that at the time echo contact with the submarine is lost the direction of her course is known within an angle of $\pm \theta$. Plate 2 shows that the horizontal limits (A_r) beyond which she cannot escape at any depth (H) are bounded by the single-hatched sector of central angle 2θ and the double-hatched areas outside this angle that are determined by her tactical turning radius (R). The sum total of these areas (A_r) proves to be:

$$A_r = U^2 \left(\theta + \frac{U}{3R}\right) \qquad (6)$$

when θ is expressed in radians, and the volume (V_r) becomes:

$$V_r = \int_0^H A_r \, dH \qquad (7)$$

Thus the required number of depth charges (N_r) becomes:

$$N_r = \frac{V_r}{V_o} N$$

$$= \frac{V_r}{V_o} \; \frac{\left(\frac{S_d \; S_s}{S_d - S_s}\right)^2 \left(3 \frac{HL^2}{S_d^2} + 3 \frac{H^2 L}{S_d S_b} + \frac{H^3}{S_b^2}\right)}{4 \; (a + B)(b + B)^2}$$

$$= \frac{\theta + \frac{U}{4R}}{\pi} \; N \quad \text{(approx.)} \qquad (8)$$

8. In the preceding paragraphs it has been shown that, whenever the speed or the course of the submarine is known with some accuracy at the time echo contact is lost, the reduced value of the theoretical number N may be found either by a simple substitution in Equation 5 or by multiplying the entire equation by a correction factor. It should be noted that in either case the basic validity of the original equation remains unimpaired. It follows, then, that the relative importance of each element or factor influencing the attack may be deduced from the part it plays in this equation toward determining the value of N.

9. From the point of view of the submarine, it becomes desirable to give N the largest possible value. Any reduction in the present values for a, b, and R, or any increase in the values of S_s and H would improve the submarine's chances for escape, but these particular factors are determined by the designer and cannot be altered except in the case of new construction. However, the fact that only 10% of the British attacks on U-boats are successful seems to indicate that the Germans recognize the truth of the old adage:

> "He who fights and runs away
> Will live to fight another day."

METHODS OF REDUCING THE NUMBER OF DEPTH CHARGES

10. Viewed from the standpoint of the destroyer, it becomes desirable to make the value of N as small as possible, since this reduces the chances of escape. An examination of Equation 5 shows that eight separate

factors are involved in the determination of the value of N. Obviously the agencies responsible for the values of these several factors are:

(a) Bureau of Ships, which (through Bureau of Construction & Repair) determines a, b, H, and S_s.

(b) Bureau of Ships, which (through the Naval Research Laboratory) determines S_d and L.

(c) Bureau of Ordnance, which determines B and S_b.

Values to be assigned to the factors in the first group need not be discussed at this point, since they are quite beyond any control by the attacker. But it will be seen that the factors S_d, L, and S_b can be improved so as to reduce greatly the value of N.

11. S_d, which is the top speed at which a destroyer can follow a submarine by means of listening and echo detection during an attack, averages something like 18 knots. Beyond this speed the local noise background masks both propeller sounds and echoes from the submarine.

12. Recent tests carried out on the USS SEMMES and the USS WILKES indicate that S_d can be increased to about 24 knots by surrounding the present projector with a dome type of shield that is relatively inexpensive and that can be quickly installed by welding it to the hull. The full effectiveness of these shields will be determined by tests to be made on the USS NOA and the USS LEARY during September. It may be noted that replacing in Equation 5 the value $S_d = 18$ by the value $S_d = 24$ reduces the value of N by over one-half.

13. The distance L, which represents the horizontal range to the submarine when echoes are first lost, depends on the depth of submergence (H), since the present standard sound projector on a destroyer, owing to its directive beam pattern, is ineffective for receiving echoes at any angle greater than α below its horizontal axis. Consequently, as the ship approaches the submarine, echo contact is lost at the range $L = \cot \alpha$. The present value of L averages about 200 yards.

14. This factor can be reduced to approximately zero by designing the sound projector so that its beam can be rotated vertically as well as horizontally. Such a design becomes practical when the projector is housed within the dome shield, and thus is relieved from the heavy strains that it otherwise must withstand. The development of such a sound projector is being vigorously prosecuted, and its design is such that it can be mounted in the standard ship's well.

15. Removing the factor L from Equation 5 further reduces the value of N to about 3/8, and contracts the equation to:

$$N = \frac{\left(\frac{S_d\ S_s}{S_d - S_s}\right)^2 \left(\frac{H^3}{S_b^2}\right)}{4\ (a + B)(b + B)^2} \qquad (9)$$

Thus it will be seen that N can be reduced to about 1/5 of its present value by adding the dome shield and making provisions for tilting the sound projector, and that, as stated, developments are well under way for providing such improvements with a minimum modification of the present standard echo detection equipment.

16. It remains to discuss the factor S_b, the velocity of fall of the bomb. A consideration of Equation 5, or its contracted form, Equation 9, shows that the velocity of fall of the depth charge appears in the denominator raised to the second power, and that it thus plays a prominent role in determining N, the number of bombs required for a successful attack. The value of S_b for the present "ash can" averages somewhere between 8 and 9 feet per second. Moreover, it has kept this same average for the past 20 years. Therefore, it can and should be stated clearly and emphatically that of all the factors that operate to determine the effectiveness of a submarine depth charge attack, one of the most important, the rate of fall of the charge itself, has received little or no consideration.

17. It appears probable that the present value of S_b can be doubled by providing a proper streamlined casing. Such a development should be undertaken without delay, because it would result in a substantial reduction in the value of N, and therefore operate strongly against the attacked submarine. But efforts should be made to increase V_b by a factor of at least 3, and as much more as possible, by making the depth charge power driven. It seems probable that a rocket drive could be made very effective for the few seconds it would be required to operate.

18. The curves of Plates 3, 4, and 5 show the numerical relation between N and each of the factors subject to our control, S_d, L, and S_b, for an attack directed against the USS PORPOISE in accordance with present procedure. These conditions evaluate the several factors of Equation 5 as follows:

S_d = 18 knots	H = 300 feet	a = 150 feet
S_s = 9 knots	L = 600 feet	b = 12.5 feet
S_b = 4.8 knots	B = 30 feet	R = 500 feet

19. Curve 1 of Plate 3 shows the relation between N and the destroyer velocity (S_d) when all other factors are held to the above values, and Curve 2 shows the relation that will hold when the factor L approximates zero as promised by developments that are under way. The value of N approaches infinity in both cases as S_d approaches 9 knots, the speed of the submarine. At a speed of 30 knots it drops to 338 and 149 respectively when L equals 600 and zero. Raising S_d from 18 knots, our present limit for effective echoing, to 24 knots decreases the value of N from 1008 to 501 when L = 600, and from 292 to 187 when L = 0. Obviously a further worth while gain would result from increasing S_d to 30 knots, but at present it appears doubtful that effective listening or echo ranging will be accomplished at speeds much higher than 24 knots.

20. The curves of Plate 4 are similar in character to those of Plate 3. Curves 1 and 2 show the relation between N and L when S_d has its present value of 18 knots and when it has the value of 24 knots, to which

it can be raised when the projector is housed within the dome shield. Curve 1, which pertains to a destroyer speed of 18 knots, shows that N drops from 1008 to 292 when the value of L is reduced from 600 to zero, and Curve 2 shows the corresponding decrease of N from 501 to 187 when the destroyer speed is raised to 24 knots.

21. The value of N as determined by the rate of fall of the depth charge (S_b) is shown by Curves 1, 2, 3, and 4 of Plate 5 for the four respective conditions:

1. $L = 600$ feet, $S_d = 18$ knots
2. $L = 600$ feet, $S_d = 24$ knots
3. $L = 0$ feet, $S_d = 18$ knots
4. $L = 0$ feet, $S_d = 24$ knots

Each of these curves shows that N decreases rapidly with any increase in the value of S_b. Using the present value of 4.8 knots (8 feet per second) as given by the present "ash can," N takes the values 1008 - 501 - 292 - and 187, respectively. The corresponding values of N when S_b is doubled and tripled become 556 - 248 - 73 - 47 and 437 - 185 - 32 - 21, respectively.

22. These curves and figures show that developments of the sound detecting and ranging apparatus that are either completed or are well under way should reduce N to about 1/5 of its present value, and that this fraction can be still further reduced to 1/50 by tripling the velocity of fall of the depth charge.

23. This represents such a marked improvement that our erstwhile theoretical N begins to take on such practical aspects as to encourage a brief consideration of attack procedure with a view to reducing it to thoroughly practical limits.

THE DIRECT HIGH SPEED ATTACK

24. It will be noted that the above numerical values of N are based on the assumption that nothing is known about the course of the submarine, and that Equation 8 shows that such knowledge reduces the value of N in accordance with the ratio of the reduced operating volume to the total operating volume. Present tactics, by taking range and bearing data, determine the course of the submarine with fair accuracy, providing she proceeds on a fixed course. But the time that intervenes between losing echo contact and completing the laying of the depth charges permits the submarine to change course to port or to starboard by approximately 90 degrees. Our knowledge of the submarine's course, therefore, cannot reduce N by a factor greater than 1/2.

25. The dome-shielded tilting projector will permit the more favorable procedure of directing the destroyer to the submarine at 24 knots without loss of detection or echo contact. During the approach, the beam tilt and azimuth bearings will give the depth H and show whether the submarine is proceeding toward port or starboard, thereby confining her course to within a 180° sector, while the neglected Attack Meter, perfected during the past year, will serve to delimit the course of the submarine to a 90° sector by indicating whether she is proceeding toward or from the destroyer.

This information confines the depth charge pattern to one quadrant of the operating circle, and thereby reduces the minimum number of depth charges (N) required to assure a successful attack to the thoroughly practical number 5. This figure, it will be remembered, is predicted on tripling the rate of fall of the present "ash can," which is about 8 feet per second. Moreover, it does not take into consideration the advantage gained through the determination of H, which limits the bomb pattern to two dimensions.

26. The advantages offered by the direct high speed attack, as outlined, have been recognized for a matter of years, as have also its attending difficulties, which proved too formidable to overcome until the advent of the dome shield, for which the British Admiralty must be credited. This shield serves the double purpose of reducing the noise background sufficiently to permit echo detection at higher speeds, and of shielding the projector from flow strains so that it can readily be designed for tilting the sound beam vertically. And, as stated, the development of such a projector is well under way.

DEPTH CHARGE PATTERNS

27. The difficulty of laying an accurately spaced depth charge pattern at the high and accelerating speed used during the attack has also been recognized and overcome through the development of the so-called "Attack Meter," which eliminates the time factor and permits directing the attack in terms of distance, even to the point of laying accurately and automatically the depth charges in a predetermined pattern. This device has been tested and favorably reported, but thus far has found no place in the anti-submarine program.

28. It remains to consider the relative merits of various types of bombs and bomb patterns that might be employed with the direct high speed anti-submarine attack. The form and dimensions of the maximum area that must be covered by the depth charge pattern, which obviously is the horizontal area enclosing the bottom of the operating volume, is shown in Plate 6 by the cross-hatched area. This figure is drawn to scale by evaluating the various factors involved as follows:

$$
\begin{aligned}
S_d &= 24 \text{ knots } (40 \text{ f.p.s.}) \\
S_s &= 9 \text{ knots } (15 \text{ f.p.s.}) \\
S_b &= 24 \text{ f.p.s.} \\
H &= 300 \text{ feet} \\
R &= 500 \text{ feet}
\end{aligned}
$$

This sets the maximum distance (U) that the submarine can run during exposure to the depth charges as 300 feet. Taking the tactical turning radius (R) of the USS PORPOISE - 500 feet - makes the breadth (m to n) of this area 174 feet. And since the axis of this area is 300 feet, it follows that the operating area of the turning center of the submarine will lie within the more simply formed sector area having the radius 300 feet and arc approximately 175 feet. Moreover, the approximate angular limits (Paragraph 25) within which this sector may lie are such as to confine it within a quadrant of the operating area. The figure shows the subject sector located in one quadrant of the operating circle.

29. The conditions to be met by the depth charge pattern are: first, that the pattern is required to cover the operating area, and not the operating volume, since the tilting beam projector permits determination of the depth (H) as the submarine is approached; and second, that a pattern which covers the maximum operating area must be designed to cover one quadrant of the operating circle at maximum depth H. This area, it will be recalled, defined the horizontal limits of the turning center of the submarine.

30. A consideration of the actual operating area (represented by the cross-hatching on Plate 6) in connection with the possible operating area (represented by the quadrant of the operating circle) shows that the submarine must run a straight radial course to reach any point of the arc, and that, therefore, the bomb pattern can be limited in a radial direction to the distance U-a, when a is the half length of the submarine and U its operating radius. This is shown on Plate 7 where the double-hatched area indicates the area to be covered by the depth charge pattern.

31. Obviously the pattern employed should be equally effective irrespective of the direction of approach. This condition is approximated by making the pattern cover a square area determined by the radial distances to be covered. Thus the number of charges required can be expressed as:

$$N = \left[\frac{U - a}{2B + 2b}\right]^2 = \left[\frac{\frac{S_d\,S_s}{S_d - S_s} \cdot \frac{H}{S_b} - a}{2B + 2b}\right]^2 \qquad (10)$$

where each factor has the same meaning as defined in Paragraph 4. Applying this formula to the USS PORPOISE gives:

$$N = \left[\frac{\frac{40 \times 15}{40 - 15} \quad \frac{300}{24} - 150}{60 + 25}\right]^2 = 3.1 \text{ bombs} \qquad (11)$$

But since our assumption requires that N must be a perfect square, the required number of bombs is 4.

32. In this connection it becomes of interest to determine the maximum speed (S_s) of the USS PORPOISE for which the 4 spot pattern would produce certain destruction. This will be found by making N equal to 4 in the formula and determining S_s. This gives for S_s a speed of 9.4 knots. Below this speed the 4 spot pattern overspreads the operating area.

33. It also becomes of interest to apply this formula to a small German submarine where a, b, H, and S_s approximate 84', 6.5', 400', and 12' per second. This gives 7.6, or a pattern of 9 as the required number of depth charges. But if the value of H is taken as 300', then the number of charges reduces to 3.2, and a 4 spot pattern will serve. The surprising increase in the number of charges required for 400' as compared with a 300' depth of submergence is due in part to the increased operating area caused by the depth itself, but still more to the fact that the border area shown

in Plate 7 forms a much smaller portion of the entire operating area. Here again we find evidence that in the design of their submarines the Germans have accentuated the factors that make for increasing the chances of escape.

34. The depth (H) for which the small German U-boats are designed is not definitely known, but reports are extant stating that they have escaped at the depth of 400 feet. Moreover, the fact that they have so successfully escaped from attacks, taken together with the further fact, as shown, that increasing the depth beyond 300 feet greatly increases the number of depth charges required, encourages a belief that these small U-boats are designed to operate at depths greater than 300 feet.

CONTACT BOMBS VS. DEPTH CHARGES

35. It remains to consider the relative merits of making the attack with contact bombs as compared with depth bombs. Obviously a contact bomb will require considerably less charge than does a depth bomb, but, at the same time, more contact bombs will be required because of the smaller effective target area. Since the ratio of the number of charges required in the two cases will approximate the reciprocal of the ratio of the effective target areas, it can be expressed as:

$ab + aB + Bb + B^2$

$$N_c = \frac{\pi (a + B)(b + B)}{\pi \, ab} N = \frac{(a + B)(b + B)}{ab} N \qquad (12)$$

$ab + B(a + b + B)$

where a and b represent the half length and the half beam of the submarine, and B the range of destruction of the depth bomb. These values, in the case of the USS PORPOISE, are 150', 12.5', and 30' respectively. Inserting these values in the equation gives $N_c = 4.1$ N. And inserting the corresponding values for the small German submarine gives: $N_c = 7.6$ N. From these relations it appears that the attack by contact bombs might effect some saving of explosive, but that the difficulty of laying the bomb pattern would be considerably greater.

36. This disadvantage may more than offset the advantage that if the contact bomb does not find the target it will not explode and render the water near the submarine opaque to sound. However, it should not be overlooked that a single line of high speed contact bombs crossing the target becomes a practical pattern. The figures given in the preceding paragraph indicate that about one depth charge out of four will land on deck in case of an attack on the USS PORPOISE, and one out of seven for an attack on the small German U-boats. Under such conditions the charges will not detonate if they are set for a depth beyond the submarine. This argues somewhat strongly that depth charges should also be provided with means for detonation upon contact with the target.

CONCLUSION

37. The purpose of this analysis and discussion of the anti-submarine attack is to evaluate the several factors that determine its success. It rates these factors in accordance with their relative effectiveness in determining the minimum number of depth charges required to eliminate the element of chance from the attack. This rating, which calls for definite modifica-

tions and improvements in the sound detecting equipment and shows the crying need for an improved depth charge, does not aim to evaluate anti-submarine tactics. However, it appears that full advantage of such improvements in equipment can be secured by the direct high-speed attack, that serious consideration should be given to the tactics involved in such an attack, and that such consideration should include the possibility of crossing the submarine with a line of high speed contact charges spaced within the beam width of the target. This simple bomb pattern should prove effective if the time interval of fall of the bomb can be made less than the time required for the target to proceed a distance equal to its length.

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TABLE 1

MEANINGS OF SYMBOLS

a = half length of submarine (in feet).
A_o = maximum operating area at depth H (in square feet).
A_r = reduced operating area at depth H (in square feet).
b = half beam of submarine (in feet).
B = destructive range of depth charge (in feet).
H = depth of submergence of submarine (in feet).
L = range of submarine at time echo contact is lost (in feet).
N = number of depth charges needed to assure success.
N_r = reduced number of depth charges needed to assure success.
N_c = number of contact bombs needed to assure success.
R = minimum tactical turning radius of submarine.
S_b = speed of fall of depth charge (in feet per second)
S_d = speed of destroyer (in feet per second).
S_s = speed of submarine (in feet per second).
U = maximum distance submarine can travel from time echo contact is lost until depth charges have reached H (in feet).
V_o = volume beyond which submarine cannot escape during attack (in cubic feet).
V_r = reduced volume beyond which submarine cannot escape during attack (in cubic feet).
V_s = volume of submarine (in cubic feet).
V_{des} = destructible volume of submarine (in cubic feet).
α = angle of dip of submarine as seen from destroyer.
θ = course angle of submarine (in radians).

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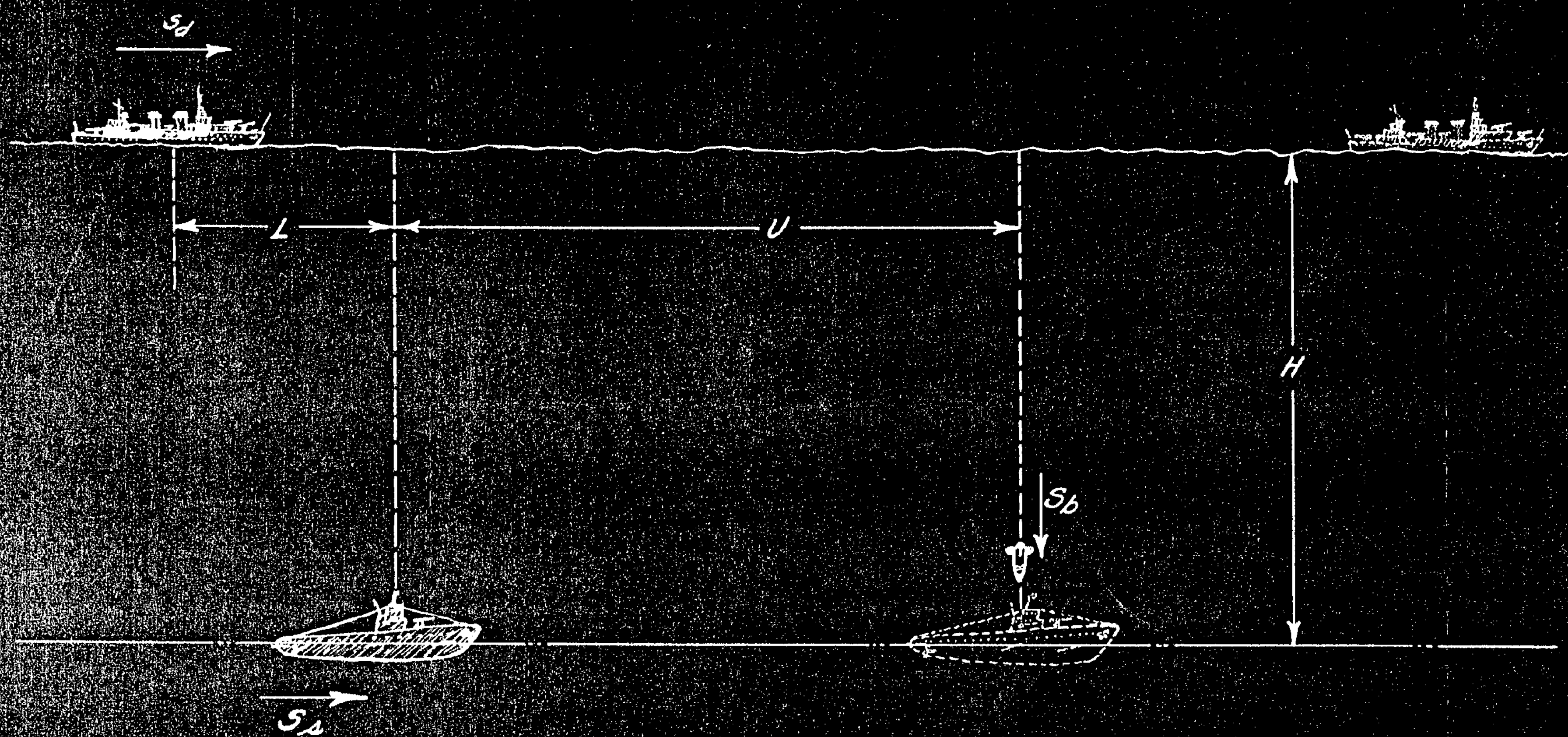
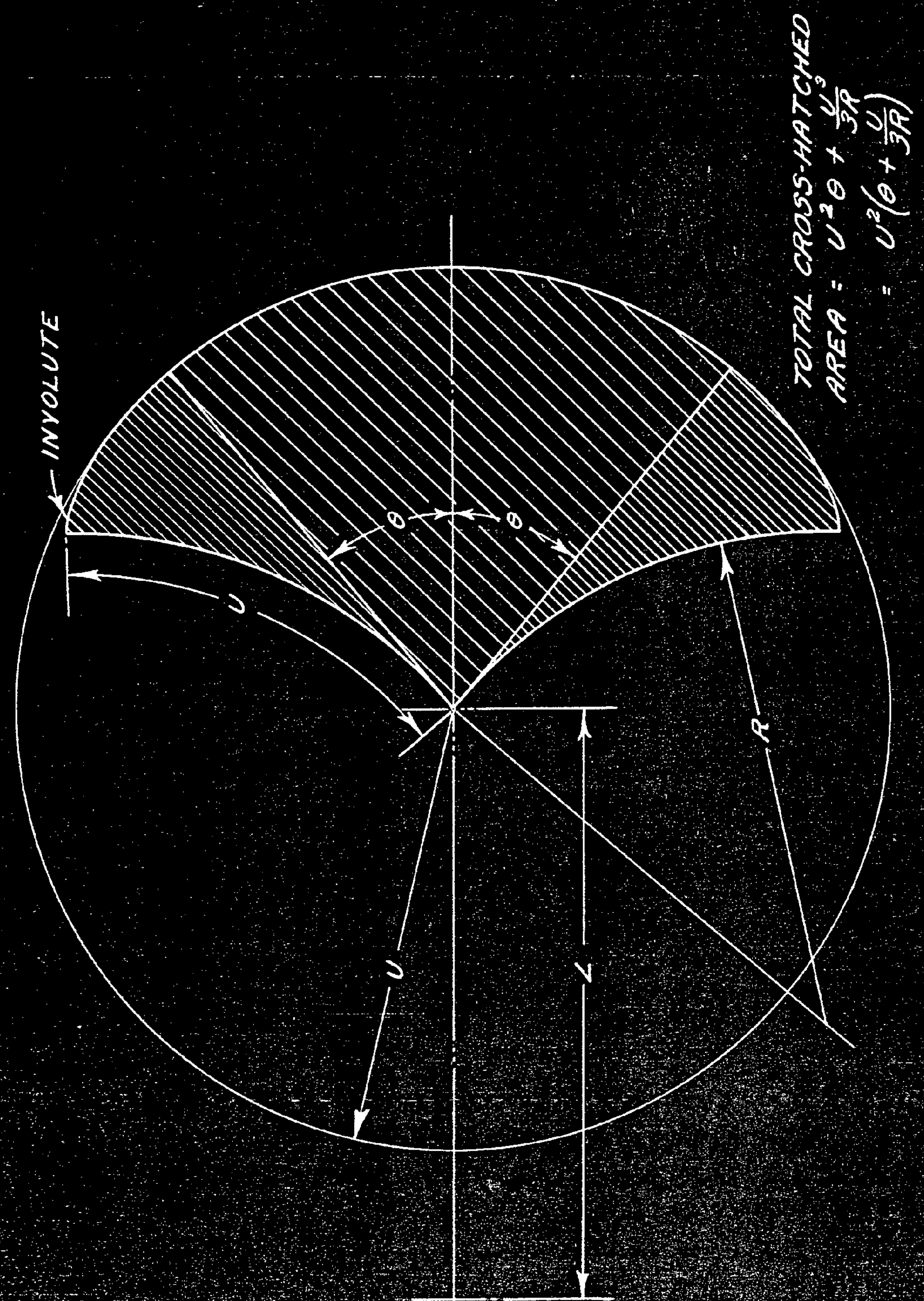


SKETCH FOR DETERMINATION OF "U"

THE TIME REQUIRED FOR THE DESTROYER TO TRAVEL THE DISTANCE $L+U$ IS $(L+U)/S_d$; THE TIME REQUIRED FOR THE DEPTH BOMB TO FALL THE DISTANCE H IS H/S_b; AND THE TIME FOR THE SUBMARINE TO TRAVEL THE DISTANCE U IS U/S_s. THEREFORE

$$\frac{L+U}{S_d} + \frac{H}{S_b} = \frac{U}{S_s}, \text{ HENCE } U = \frac{S_d S_s}{S_d - S_s}\left(\frac{L}{S_d} + \frac{H}{S_b}\right)$$

PLATE I.



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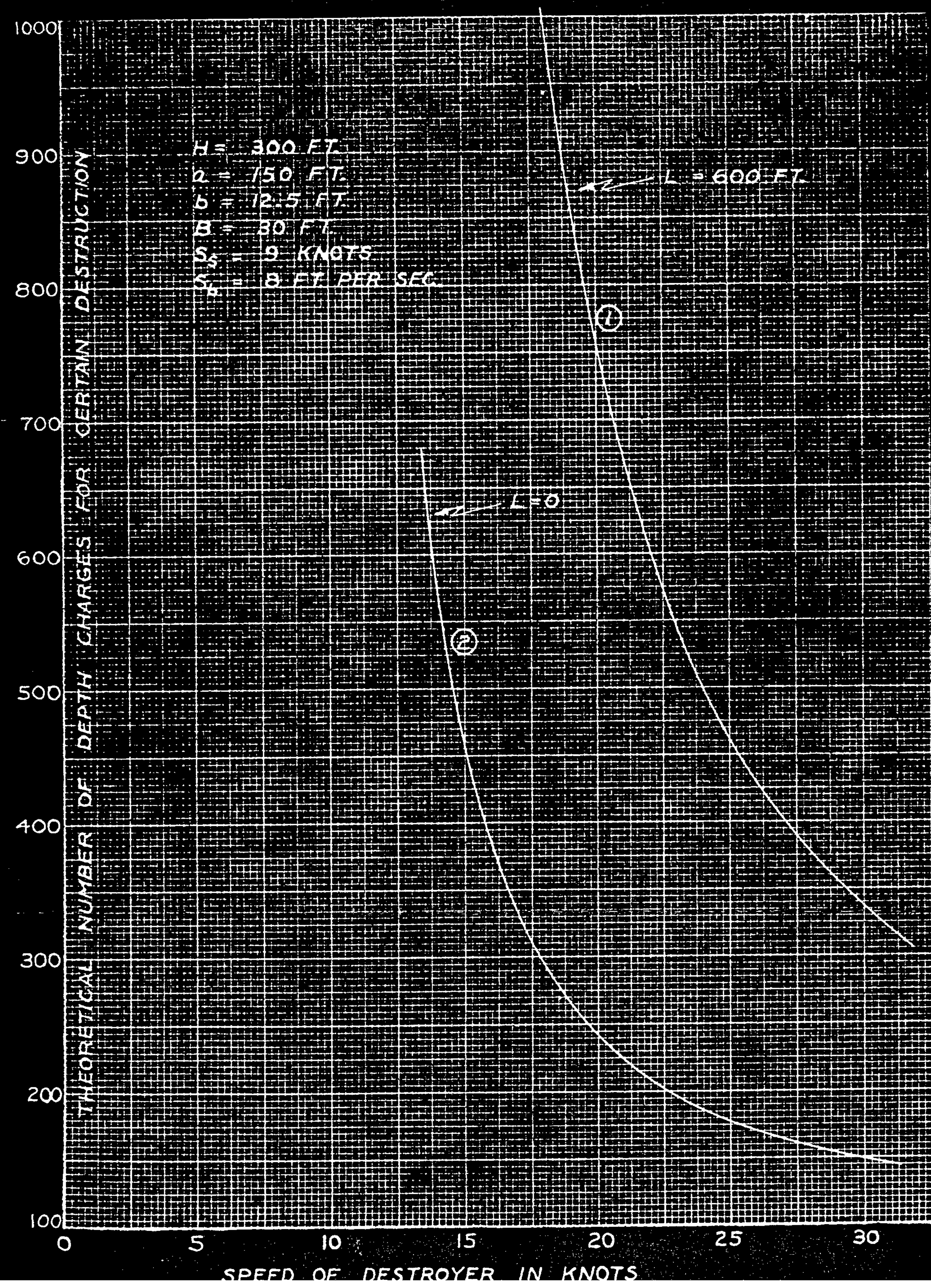

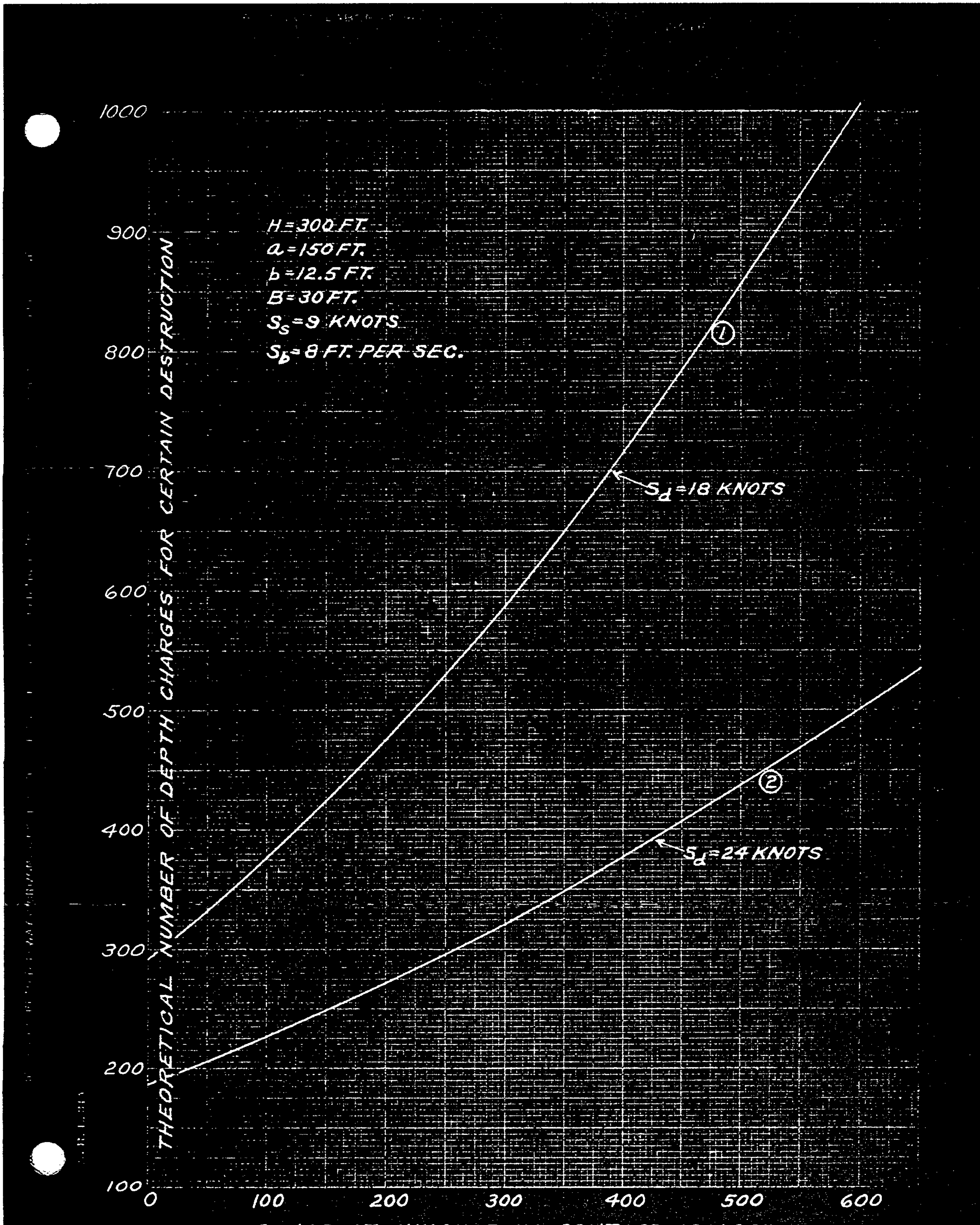
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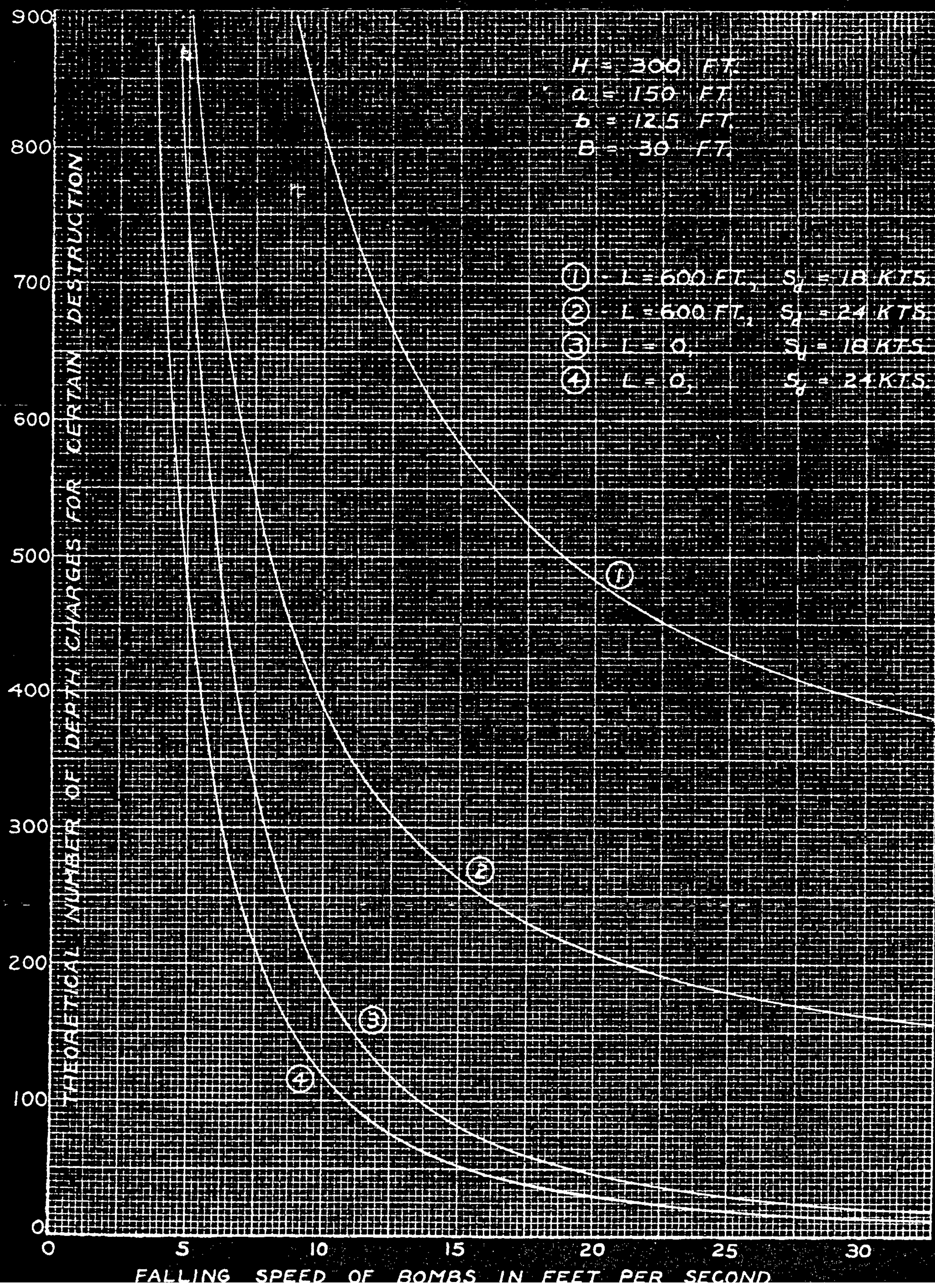
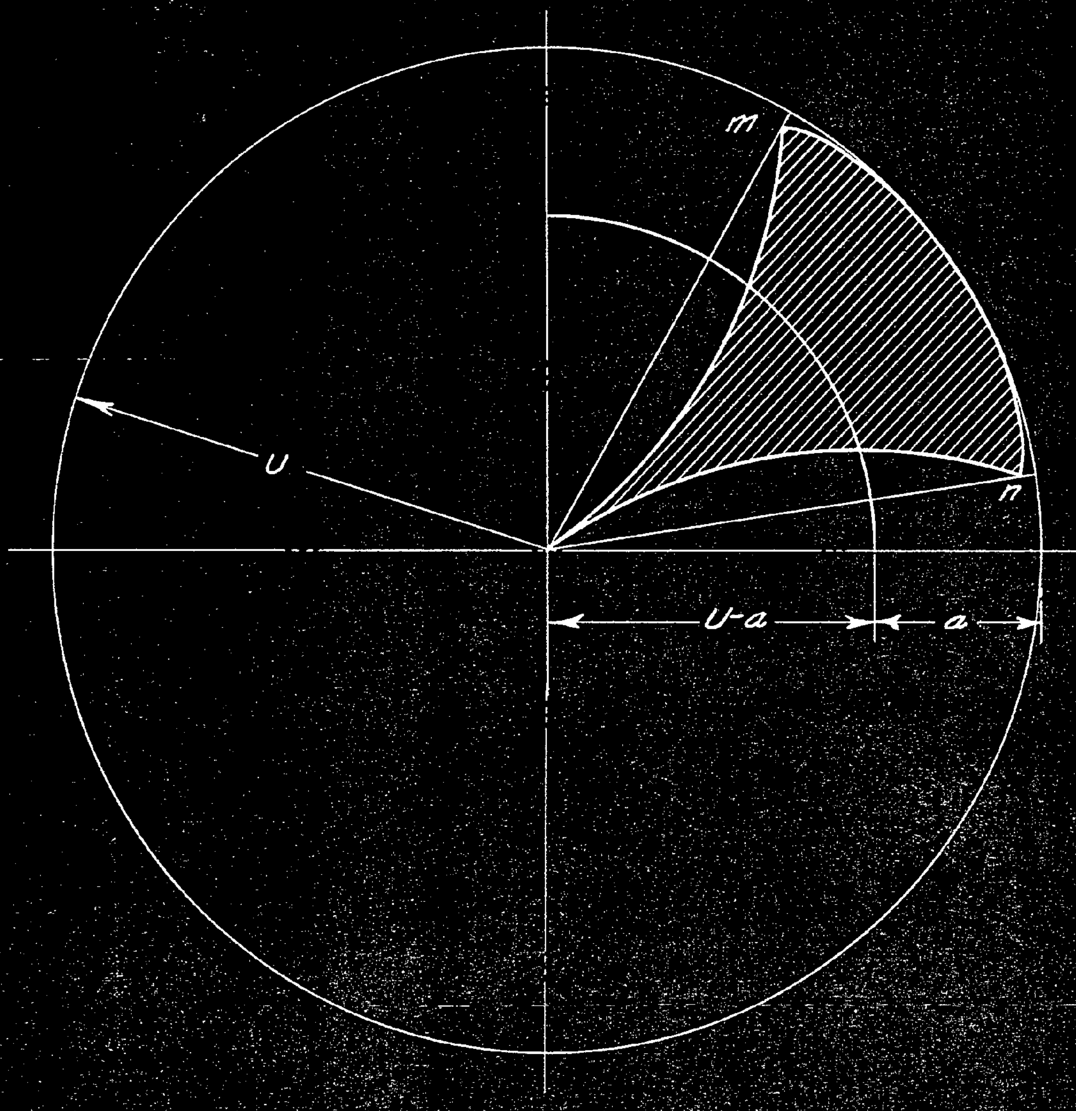
b = 12.5 FT.
B = 30 FT.
S_s = 9 KNOTS
S_b = 8 FT. PER SEC.
1
S_d = 18 KNOTS
2
S_d = 24 KNOTS
THEORETICAL NUMBER OF DEPTH CHARGES FOR CERTAIN DESTRUCTION
RANGE AT WHICH ECHO CONTACT IS LOST

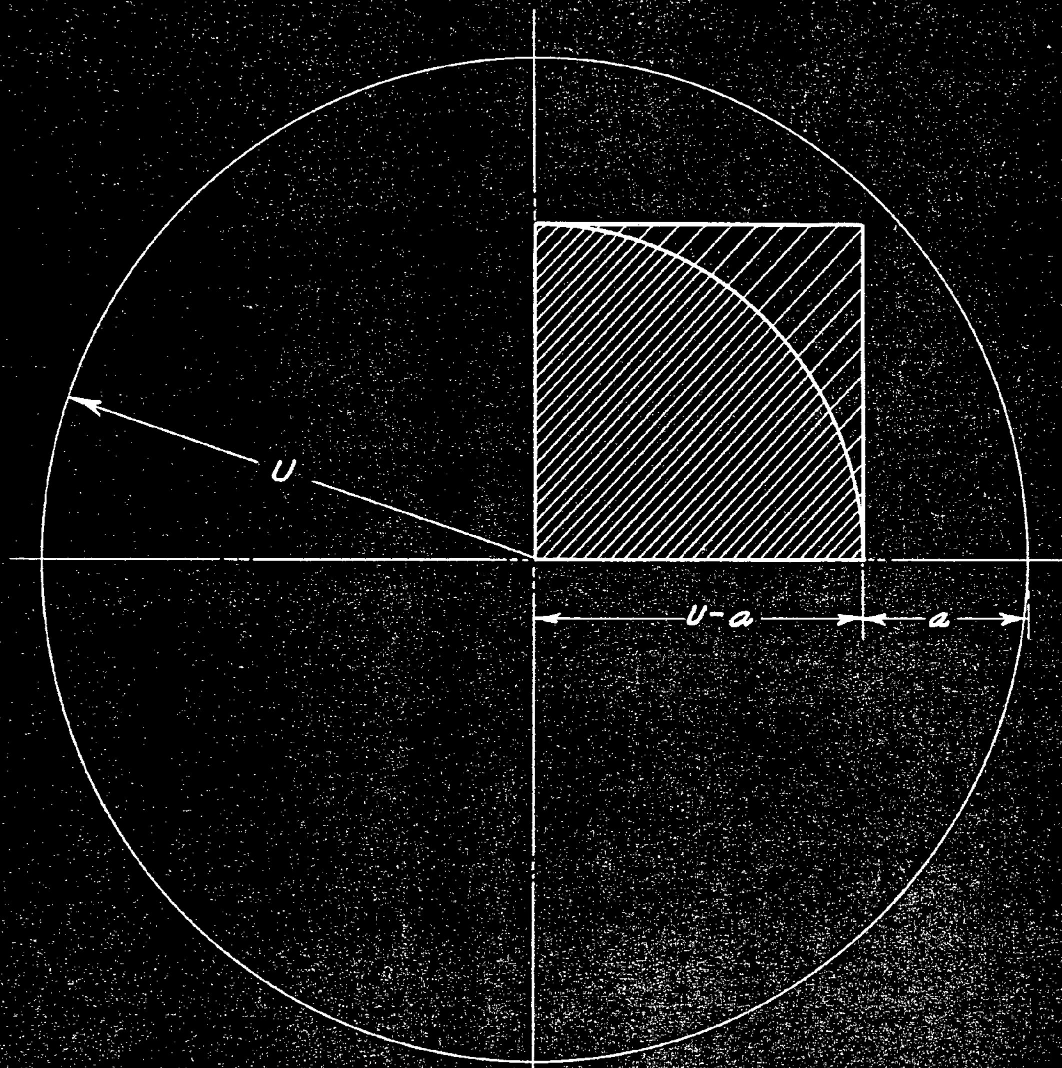


H = 300 FT.
a = 150 FT.
b = 12.5 FT.
B = 30 FT.
① - L = 600 FT., Sd = 18 KTS.
② - L = 600 FT., Sd = 24 KTS.
③ - L = 0, Sd = 18 KTS.
④ - L = 0, Sd = 24 KTS.
THEORETICAL NUMBER OF DEPTH CHARGES FOR CERTAIN DESTRUCTION
FALLING SPEED OF BOMBS IN FEET PER SECOND
0
100
200
300
400
500
600
700
800
900
5
10
15
20
25
30



OPERATING QUADRANT FOR
DIRECT ATTACK

PLATE 6



BOMB PATTERN AREA DURING
DIRECT ATTACK

PLATE 7